Microgravity Sci. Technol. (2013) 25:121–126
DOI 10.1007/s12217-012-9336-3

ORIGINAL ARTICLE

Particle Dynamics in a Fluid Under High Frequency Vibrations of Linear Polarization

D. V. Lyubimov · A. Y. Baydin · T. P. Lyubimova

Received: 30 November 2011 / Accepted: 14 December 2012 / Published online: 8 January 2013

Abstract The paper deals with the investigation of a behavior of a system of particles suspended in a fluid in a container subjected to high frequency translational vibrations of linear polarization. Pair interaction forces act on the particles under these conditions. These forces decrease with the distance and depend on the interacting particles orientation with respect to the vibration direction. The presence of these forces leads to the formation of structures in space. The problem is solved numerically using molecular dynamics method with pair interaction approximation. It is shown that the process of the structures formation consists of the fast stage of compact cluster formation and slow evolution of these clusters. It is found that for vibrations of linear polarization the particles form the chains oriented perpendicular to the direction of vibrations. At long time-scales these chains form the layers perpendicular to the direction of the vibrations and located almost periodically all over the fluid volume.

Keywords Particles · Vibrations · Structure formation · Ordering

D. V. Lyubimov · A. Y. Baydin
Perm State University, 15, Bukirev str., 614990, Perm, Russia

D. V. Lyubimov
e-mail: lyubimov@psu.ru

T. P. Lyubimova (✉)
Institute of Continuous Media Mechanics UB RAS,
1, Koroleva Str., 614013, Perm, Russia
e-mail: lubimova@psu.ru

Introduction

It is known that vibrations are able to exert strong influence on the behavior of inhomogeneous hydrodynamic systems. For instance, inclusions having higher density than density of the surrounded fluid can float up, and sink if their density is less (Lugovtsov and Sennitsky 1986; Lyubimov et al. 1987). In case of deformable particles linear polarized vibrations flatten them in the direction of vibrations (Lyubimov et al. 1996). In a non-uniform pulsational flow average force acts to the particles suspended in a fluid. Flow non-uniformity can be related to the presence of other particles, in this case effective interaction of particles arises under vibrations (Lyubimov et al. 1992, 2001). These interaction forces diminish with the increase of distance between the particles and depend on the orientation of interacting particles pairs with respect to the vibration direction. At small distances the viscous effects start to play important role which may even lead to the change of the force sign (Klotsa et al. 2007, 2009; Lyubimova et al. 2008, 2011). Such complicated dependence of interaction force on the distance and orientation may lead to the far distance order in the system of particles suspended in the oscillating fluid. In the present paper numerical investigation of the evolution of ensemble of particles in a fluid subjected to the high frequency vibrations of linear polarization is performed.

Interaction of Two Particles

Let us consider the interaction of two similar rigid particles in a fluid subjected to a uniform imposed pulsational flow. Gravity field is absent. We assume that the imposed pulsational flow frequency ω is such high that the thickness of Stokes boundary layer $\delta = (\nu/\omega)^{1/2}$ near the particle surfaces is small in comparison with the average distance between the particles and with the particle size. This allows to consider the flow as inviscid everywhere except for the vicinity of particles. At the same time, vibrations are assumed to be non-acoustic which allows to consider the fluid as incompressible. Equations for the pulsational flow can be linearized if the vibration amplitude is small in comparison with the average distance between the particles and with the particle size. In terms of non-dimensional parameters this means that the Reynolds number is small in comparison with the dimensionless vibration frequency (Lyubimova et al. 2011).

Pulsational field around a particle is non-uniform due to the presence of second particle. In the leading order, this field can be represented as the sum of uniform imposed pulsational field $\vec{v}_\infty$ and non-uniform pulsational field $\vec{v}_r$ scattered on the second particle:

$$\vec{v} = \vec{v}_\infty + \vec{v}_r$$

Velocity of a particle motion in oscillating fluid can be represented as

$$\vec{u} = \mathrm{Re}\big(\vec{U} e^{i\,\omega t}\big)$$

According to the assumptions introduced above, the flow is considered as potential $\vec{v}_r = \nabla\phi$, thus it is determined from the following equation and boundary conditions

$$\Delta\phi = 0, \qquad \partial\phi/\partial n|_s = u_n \tag{1}$$

or in terms of pulsational flow potential amplitude Φ related to ϕ as $\phi = \mathrm{Re}\left(\Phi e^{i\,\omega t}\right)$:

$$\Delta\Phi = 0, \qquad \partial\Phi/\partial n|_s = U_n \tag{2}$$

The solution to the Laplace equation (2) is sought in the form

$$\Phi = \frac{\vec{A}\vec{r}}{r^3}\,. \tag{3}$$

(dipole approximation).

Satisfying the boundary conditions we obtain for $\vec{v}_r$

$$\vec{v}_r = R^3\frac{\rho_s - \rho}{\rho + 2\rho_s}\left(\frac{\vec{v}_\infty}{r^3} - 3\frac{\vec{v}_\infty\cdot\vec{r}}{r^5}\vec{r}\right) \tag{4}$$

Here it is taken into account that the velocity of spherical particle subjected to a uniform imposed pulsational field is (Landau and Lifshitz 1987):

$$\vec{u} = \frac{3\rho}{\rho + 2\rho_s}\vec{v}_\infty \tag{5}$$

To determine the energy of the pulsational field we need to calculate the squared pulsational velocity

$$v^2 = v_\infty^2 + \ 2\vec{v}_\infty\cdot\vec{v}_r\ + v_r^2 \tag{6}$$

In Eq. 6 it is sufficient to take into account only interferential term since the first term is spatially uniform, therefore it does not give any contribution into the force and the last term is of the higher order of smallness with respect to the ratio of particle size to the distance to this particle.

Thus, we have for the squared pulsational velocity:

$$v^2 = 2R^3\frac{\rho_s - \rho}{\rho + 2\rho_s}\left(\frac{v_\infty^2}{r^3} - 3\frac{(\vec{v}_\infty\cdot\vec{r})^2}{r^5}\right) + \ldots \tag{7}$$

and for its gradient:

$$\nabla v^2 = 6R^3\frac{\rho_s - \rho}{\rho + 2\rho_s} \times\left(-\frac{v_\infty^2}{r^5}\vec{r} + 5\frac{(\vec{v}_\infty\cdot\vec{r})^2}{r^7}\vec{r} - 2\frac{(\vec{v}_\infty\cdot\vec{r})\,\vec{v}_\infty}{r^5}\right) \tag{8}$$

Taking into account that $\vec{v}_\infty = \vec{j}\,a\omega\cos\omega t$, introducing the unit vector $\vec{\tau} = \vec{r}/r$ in the direction from the particle under consideration to the other particle and the angle ϑ between the axis of vibrations and the line connecting the particles $\vec{j}\cdot\vec{\tau} = \cos\vartheta$ and averaging over vibration period we obtain

$$\nabla\overline{v^2} = 3a^2\omega^2R^3\frac{\rho_s - \rho}{\rho + 2\rho_s}\left(\frac{5\cos^2\vartheta - 1}{r^4}\vec{\tau} - 2\frac{\cos\vartheta}{r^4}\vec{j}\right) \tag{9}$$

In a particular case of tangential vibrations ($\vartheta = 0$) we have

$$\nabla\overline{v^2} = 6a^2\omega^2R^3\frac{\rho_s - \rho}{\rho + 2\rho_s}\frac{\vec{\tau}}{r^4}, \tag{10}$$

and in the case of normal vibrations ($\vartheta = \pi/2$):

$$\nabla\overline{v^2} = -3a^2\omega^2R^3\frac{\rho_s - \rho}{\rho + 2\rho_s}\frac{\vec{\tau}}{r^4}, \tag{11}$$

The average force acting to a spherical particle in a non-uniform pulsational velocity field was calculated using assumptions formulated above in Krasilnikov and Krylov (1984) (see also Lyubimov et al. 2002):

$$\vec{F} = \frac{3}{4}\mathrm{V}\rho\frac{\rho_s - \rho}{\rho_s + \frac{1}{2}\rho}\nabla\overline{v^2} \tag{12}$$

Here ρ is the fluid density, ρ_s is the density of particle, V is the particle volume. This formula does not take into account the viscosity, thus it is valid for large enough distances from the particle (large in comparison with the Stokes boundary layer thickness).

As one can see from Eqs. 9–12, for large enough distances between the particles we have repulsion for tangential vibrations and attraction for normal vibrations. For arbitrary angles between vibration axis and the line connecting the particles, the interaction force component normal to the line connecting the particles is non-zero. It turns the particle pair.

At small distances the viscous deceleration of pulsational flow between the particles is at work. It leads to the change from attraction to repulsion for normal vibrations (Klotsa et al. 2007, 2009; Lyubimova et al. 2011, 2008) and contrary to that for the tangential vibrations (Lyubimova et al. 2008).

Ensemble of Particles

Let us consider now ensemble of particles. Each particle is subjected to the influence of the pulsational fields of all the other particles. In the leading order with respect to the ratio of particle size to the distance between the particles it is sufficient to take into account interferential terms of the imposed flow and pulsational field scattered on the particles. This means that in the calculations of interaction force it is possible to implement pair-wise approximation.

In this approximation, for the inviscid case, using Eqs. 9 and 12, we obtain for the force acting to the i−th particle:

$$\vec{F}_i = \sum_k \vec{f}_{ik}, \quad \vec{f}_{ik} = -\frac{3\, m^i_{eff} C W^2}{r^4_{ik}} \times \left[\vec{\tau}_{ik} + 2\left(\vec{j}\cdot\vec{\tau}_{ik}\right)\vec{j} - 5\left(\vec{j}\cdot\vec{\tau}_{ik}\right)^2 \vec{\tau}_{ik} \right] \tag{13}$$

where $W = a\omega$ is the amplitude of the velocity of the imposed pulsational flow,

$$\vec{\tau}_{ik} = \frac{\vec{r}_{ik}}{r_{ik}}, m^i_{eff} = \frac{3}{2}\rho V \frac{\rho_s - \rho}{\rho_s + 0.5\rho}, C = \frac{R^3}{2}\frac{\rho_s - \rho}{\rho_s + 0.5\rho}$$

Let us now account for the viscosity. It is convenient to decompose the force of pair interaction into two orthogonal components: radial component $\vec{f}_t$ parallel to the vector $\vec{\tau}$ and responsible for the change of distance between the particles and the component $\vec{f}_n$ orthogonal to $\vec{f}_t$ and responsible for the change of orientation of particle pair.

Taking into account the asymptotics (13) for large distances between the particles and the results (Klotsa et al. 2007, 2009; Lyubimova et al. 2008, 2011) on average force sign change at the distances of the order of Stokes length, we introduce the following approximation for $\vec{f}_t$ and $\vec{f}_n$:

$$\vec{f}_t = 3\, m^i_{eff} C W^2 R_t(r)\; T_t(\theta)\; \vec{\tau},$$

$$R_t(r) = \frac{\alpha\,(r-a)}{r^5 + \beta\,(r+\varepsilon)}, T_t(\theta) = 1 - 3\cos^2(\theta) \tag{14}$$

$$\vec{f}_n = 3\, m^i_{eff} C W^2 R_n(r)\; T_n(\theta)\; \vec{n},$$

$$R_n(r) = \left\{ \frac{\alpha\,(r-a)}{r^5 + \beta\,(r+\varepsilon)}, r > r_*, \frac{\alpha\,(r_* - a)}{r^5_* + \beta\,(r_* + \varepsilon)}, r \le r_* \right\},$$

$$T_n(\theta) = \sin(\theta)\cos(\theta) \tag{15}$$

where r_*, α, a, β, ε are empirical parameters.

Equations of motion for each particle are the following:

$$m\ddot{\vec{R}}_i = \vec{f}_i - b\dot{\vec{R}}_i, \quad i = \overline{1, N}, \tag{16}$$

where b is the dissipation coefficient. The second term on the right hand side of Eq. 16 is responsible for the viscous friction force and $\vec{f}_i$ is given by Eqs. 14–15.

Numerical Simulation

Numerical modeling of the dynamics of the ensemble of particles with the interaction forces introduced in the previous section was carried out by the molecular dynamics method. We performed numerical integration of the Eq. 16. The particles were considered as similar. The interaction of the particles with the container walls was neglected. The calculations were performed for two-dimensional case. Parameters $r_* = 1.2 \cdot R$, $\alpha = 40\ \beta\varepsilon/a$, $a = 0.3$, $\beta = (2-5a)/16\,(a+\varepsilon)$, $\varepsilon = 0.5$ were adjusted with respect to the results (Klotsa et al. 2007, 2009; Lyubimova et al. 2008, 2011).

Two variants of initial distributions of particles over the computational domain were considered: random and compact.

For random initial distribution of particles numerical calculations were carried out for fixed values of the vibration amplitude $a = 0.01$ and particle radius $r_0 = 0.5$; vibration frequency was varied in the range $\omega = 1{,}000 \div 4{,}000$, the dissipation coefficient in the range $b = 0.2 \div 1$. The number of particles was 100. The calculations show that temporal evolution of the particle ensemble from the initial state consists of two stages: fast formation of the clusters including small number of particles and very slow evolution of these clusters. Each cluster forms the chain orthogonal to the direction of vibrations, then clusters gradually create the layers separated from each other by the zones free of particles. Duration of the first stage decreases with the increase of vibration frequency and with the decrease of dissipation coefficient. The larger number of particles in the volume the faster cluster formation is observed. Typical steps of the first stage of the process are shown in Fig. 1. Numerical results well correspond to the experimental results (Klotsa et al. 2007). The second, slow, stage was not analyzed for random initial distribution of particles because of time consuming.

For the compact initial distribution the calculations were performed at fixed values of all parameters: $a = 0.01$, $\omega = 2{,}000$, $r_0 = 0.5$, $b = 0.6$. The number of particles was taken to be 30. In this case it was possible to study the particles dynamics up to the large enough time-scales. The results are presented in Fig. 2. The calculations show that after formation of clusters, the slow process of there motion and joining

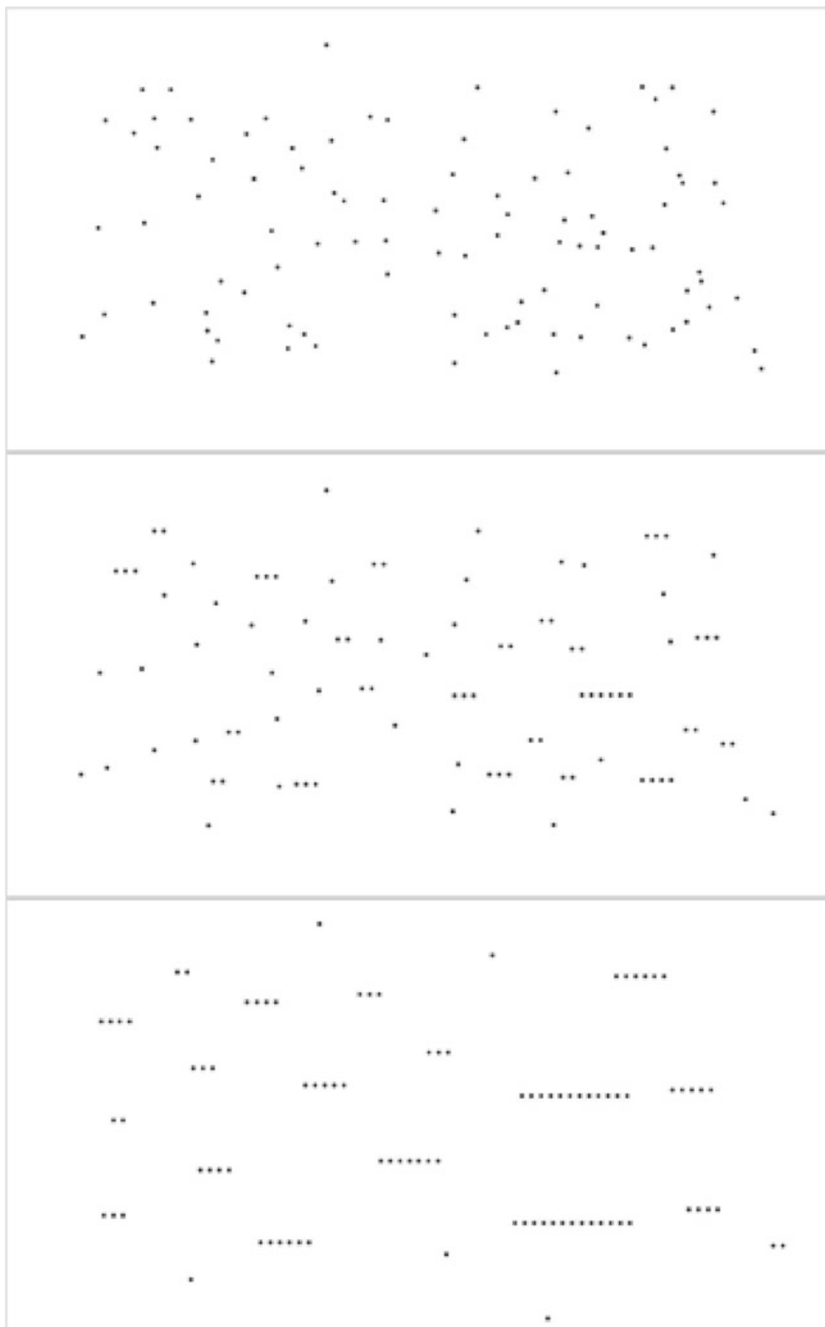

Fig. 1 Temporal evolution from random initial distribution. *Upper picture*—initial distribution, *middle*—stage of cluster formation, *lower*—formation of layers from clusters. $N = 100$, $a = 0.01$, $\omega = 2{,}000$, $r_0 = 0.5$, $b = 0.6$. Pictures are taken at $t = 0$, $t = 200$, $t = 7{,}830$

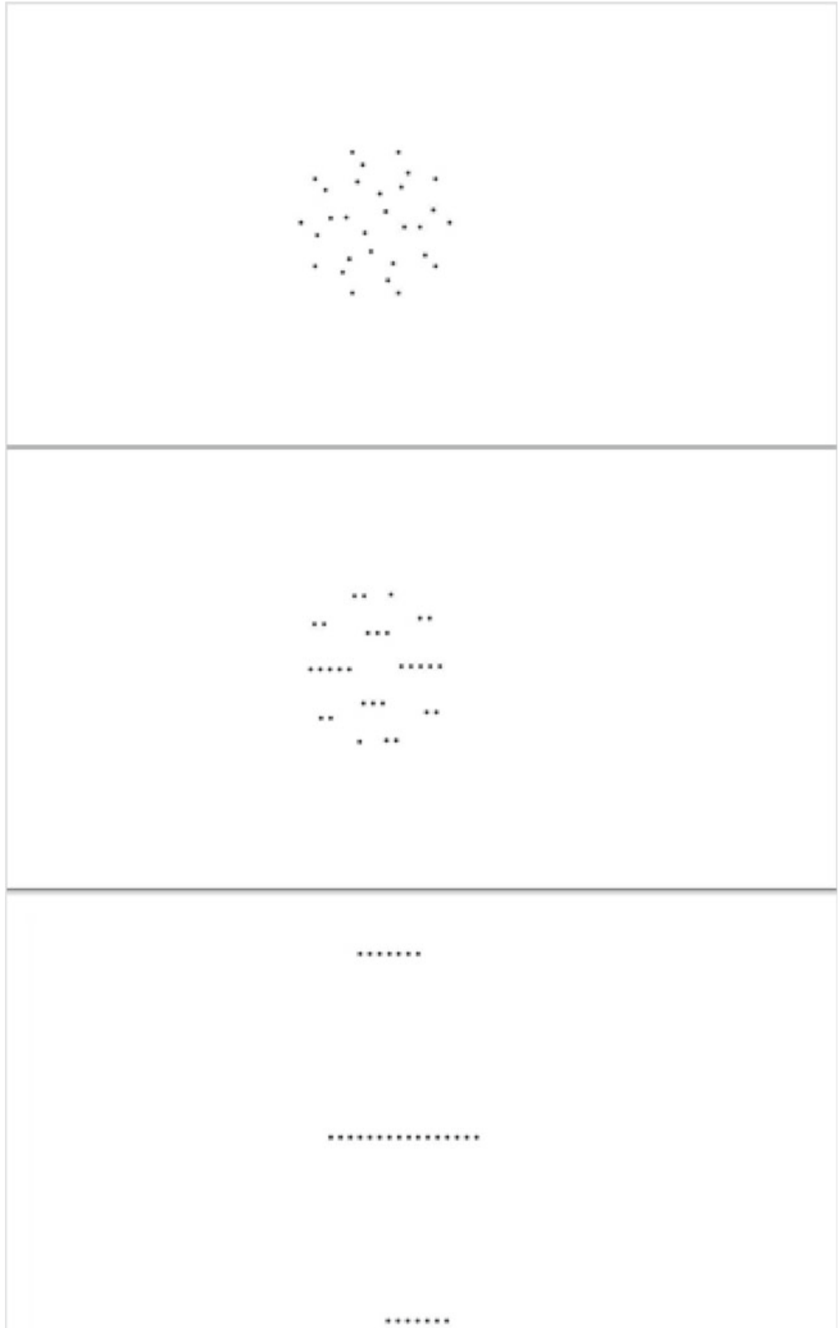

Fig. 2 Temporal evolution from a compact initial distribution. *Upper picture*—initial distribution, *middle*—stage of cluster formation, *lower*—formation of layers from clusters and growth of the size of domain including the particles. $N = 30$, $a = 0.01$, $\omega = 2{,}000$, $r_0 = 0.5$, $b = 0.6$. Pictures are taken at $t = 0$, $t = 50$, $t = 25{,}000$

is observed. This process is accompanied by the increase of distance between the particle layers.

Conclusions

- Vibrations lead to the formation of structures in the ensemble of particles suspended in a liquid.
- Under high frequency vibrations of linear polarization the particles are concentrated in the layers orthogonal to the vibration direction.
- Pattern formation discussed in the present paper reflects the existence of orienting effect of high frequency vibrations discussed for the first time in Lyubimov et al. (1997).

References

Klotsa, D., Swift, M.R., Bowley, R.M., King, P.J.: Interaction of spheres in oscillatory fluid flows. Phys. Rev. E **76**, 056314 (2007)

Klotsa, D., Swift, M.R., Bowley, R.M., King, P.J.: Chain formation of spheres in oscillatory fluid flows. Phys. Rev. E **79**, 021302 (2009)

Krasilnikov, V.A., Krylov, V.V.: Introduction to Physical Acoustics, Nauka, Moscow (1984) (in Russian)

Landau, L.D., Lifshitz, E.M.: Fluid mechanics. Course Theor. Phys. **6**, 552 (1987)

Lugovtsov, V.A., Sennitsky, V.L.: Motion of a body in a vibrating liquid. Soviet Physics. Doklady **31**(7), 530 (1986)

Lyubimov, D., Cherepanov, A., Lyubimova, T.: Behavior of a drop (bubble) in a non-uniform pulsating flow. Adv. Space Res. **29**(4), 667 (2002)

Lyubimov, D.V., Lyubimova, T.P., Cherepanov, A.A.: On a motion of solid body in a vibrating fluid. In: Zhukhovitsky, E.M. (ed.) Convective Flows, pp. 61–70 (1987)

Lyubimov, D.V., Cherepanov, A.A., Lyubimova, T.P.: The motion of solid body in a liquid under the influence of a vibration field. In: Reviewed Proc. of the First Int. Symp. on Hydromechanics and Heat/Mass Transfer in Microgravity, pp. 247–251. Gordon and Breach (1992)

Lyubimov, D.V., Cherepanov, A.A., Lyubimova, T.P., Roux, B.: Deformation of gas or drop inclusion in high frequency vibrational field. Microgravity Q. **6**(2–3), 69 (1996)

Lyubimov, D.V., Cherepanov, A.A., Lyubimova, T.P., Roux, B.: Orienting effect of vibrations on the interfaces. C.R.A.S. **325**(II b), 391 (1997)

Lyubimov, D.V., Cherepanov, A.A., Lyubimova, T.P., Roux, B.: Vibration influence of a two-phase system in weightlessness conditions. J. Phys. IV **11**(Pr6), 83 (2001)

Lyubimova, T., Cherepanova, A., Lyubimov, D.: Behaviour of drops and bubbles in non-uniform pulsational flows. In: Danier, J., Finn, M.D., Mattner, T. (eds.) XXII International Congress of Theoretical and Applied Mechanics (ICTAM 2008, Adelaide), p. 173. Abstracts book (2008)

Lyubimova, T., Lyubimov, D., Mikhail Shardin, M.: Interaction of rigid cylinders in a low Reynolds number pulsational flow. Microgravity Sci. Technol. **23**(3), 305 (2011)